Assignment 2

Numerical Methods, 2026 Spring

Due on April 16, 10:00 AM

Note: you should explain how you obtain your solution in your submission. If you use MATLAB or any other software to compute your results, you should provide your code AND describe your solving process. This is a good practice for you to explain things in a logical, organized, and concise way! **Please upload your assignment to the E3 course website.**

1. (15%) Solve the following system by Gaussian elimination with partial pivoting. State the column(s) in which row interchanges are required.

$$\begin{bmatrix} 4 & 2 & -2 & -1 \\ 0 & 4 & 1 & 2 \\ 3 & -2 & 1 & 2 \\ 2 & 0 & 3 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 7 \\ 10 \\ 2 \\ 3 \end{bmatrix}.$$

2. (25%) Use scaled partial pivoting to solve:

$$\begin{bmatrix} 4.13 & -2.20 & 0.95 & 3.02 \\ 6.14 & 4.45 & -1.45 & -4.02 \\ 1.03 & 1.86 & 0.44 & 5.22 \end{bmatrix}.$$

   (a) Employ six significant digits.

   (b) Repeat with only three significant digits. Is the solution much different?

3. (30%) The system

$$\begin{bmatrix} 4 & -1 & 0 & 0 & 0 & 0 & 100 \\ -1 & 4 & -1 & 0 & 0 & 0 & 200 \\ 0 & -1 & 4 & -1 & 0 & 0 & 200 \\ 0 & 0 & -1 & 4 & -1 & 0 & 200 \\ 0 & 0 & 0 & -1 & 4 & -1 & 200 \\ 0 & 0 & 0 & 0 & -1 & 4 & 100 \end{bmatrix}$$

   is an example of a symmetric matrix. Because the elements at opposite positions across the diagonal are exactly the same, it can be stored as a matrix with n rows but only three columns.

(a) Write an algorithm for solving a symmetric tridiagonal system that takes advantage of such compacting.

(b) Use the algorithm from part (a) to solve the system.

(c) How many arithmetic operations are needed with this algorithm for a system of N equations?

4. (40%) Solve this system with the Jacobi method. First rearrange to make it diagonally dominant if possible. Use $[0, 0, 0]$ as the starting vector.

$$\begin{bmatrix} 7 & -3 & 4 & 6 \\ -3 & 2 & 6 & 2 \\ 2 & 5 & 3 & -5 \end{bmatrix}.$$

(a) Solve this system with the Jacobi method.

(b) Solve this system with the Gauss–Seidel method.